Problem 10957. (Proposed by Victor Alexandrov, Sobolev Institute of Mathematics, Novosibirsk, Russia). Let $S^2$ be a unit sphere in $\mathbb{R}^3$. Let $D$ be a domain in $S^2$ with piece-wise smooth boundary. Let $\hat{N}$ denote the function on the sphere that maps each point to the unit inward normal vector at that point. Let $\hat{n}$ denote the function on the smooth part of the boundary $\partial D$ of $D$ that maps each such point to the inward unit vector normal to $\partial D$, and parallel to the plane tangent to the sphere at that point. Let $\sigma$ be the usual measure on a sphere, and let $s$ be the arc length measure on the boundary of $D$. Prove that

$$2 \iint_D \hat{N} \, d\sigma + \int_{\partial D} \hat{n} \, ds = 0.$$