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'Problem and Solution' section.

Problem 10957. (*Proposed by Victor Alexandrov, Sobolev Institute of Mathematics, Novosibirsk, Russia*). Let S^2 be a unit sphere in \mathbb{R}^3 . Let D be a domain in S^2 with piece-wise smooth boundary. Let \hat{N} denote the the function on the sphere that maps each point to the unit inward normal vector at that point. Let \hat{n} denote the function on the smooth part of the boundary ∂D of D that maps each such point to the inward unit vector normal to ∂D , and parallel to the plane tangent to the sphere at that point. Let σ be the usual measure on a sphere, and let s be the arc length measure on the boundary of D . Prove that

$$2 \iint_D \hat{N} d\sigma + \int_{\partial D} \hat{n} ds = 0.$$