ALGEBRAS OF MONSTER TYPE AND THE DOUBLE AXIS CONSTRUCTION

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Axial algebras are commutative non-associative algebras generated by special semi-simple idempotents called axes. Classes of axial algebras are distinguished by the fusion rules which restrict multiplication of eigenvectors with respect to the adjoint action of an axis. When fusion rules are graded, every axis leads to a group of automorphisms of the algebra, and in this way, axial algebras are inherently related to groups. The motivating examples of axial algebras are the Jordan algebras, corresponding to classical and some exceptional groups, as well as the $196,884$-dimensional real Griess algebra for the Monster sporadic simple group. The axioms of axial algebras descend from the Majorana algebras of Ivanov [1]. It is hoped that within the paradigm of axial algebras we can build a theory involving all or most of the finite simple groups.

The class of algebras of Jordan type η was introduced by Hall, Rehren and Shpectorov in [2], where it is proved (see also [3]) that, for $\eta \neq \frac{1}{2}$, the only algebras arising are the Matsuo algebras corresponding to the 3-transposition groups. The case $\eta = \frac{1}{2}$, where the Jordan algebras arise, remains open.

The class of algebras of Monster type generalizes both the algebras of Jordan type and the Griess algebra. Until recently, the Griess algebra, its known subalgebras and a few individually computed algebras for small groups were the only known additions. In the talk we will discuss the double axis construction which is a rich source of examples of algebras of Monster type. It originated from the GAP computations performed by Galt, Mamontov and Staroletov, the $3A$, and it was then developed by Joshi [4] and by Galt, Joshi, Mamontov, Shpectorov and Staroletov in [5], where, in particular, the algebras arising from the symmetric groups were determined. More recently, Hoffman, Rodrigues and Shpectorov [6] completed the case of the unitary groups over the field with four elements.

References

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