MALCEV'S PROBLEMS, WEAK SECOND ORDER LOGIC, AND BI-INTERPRETABILITY

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Malcev's problems on definable subgroups of a free non-abelain group F were solved a few years ago by Kharlampovich and Miasnikov and also by Perin, Pillay, Sklinos, and Tent.

It turned out that only cyclic subgroups are definable proper subgroups of F. Similar results hold for torsion-free hyperbolic groups. On the other hand, in finitely generated abelian groups only subgroups of finite index and the trivial subgroup are definable. One may consider the following question: what are finitely generated infinite groups where all finitely generated subgroups are definable? Furthermore, are there any interesting infinite groups G where finitely generated subgroups are uniformly definable, i.e., for each natural n there exists a first-order formula $D_n(x, y_1, \ldots, y_n)$ such that for any elements $g_1, \ldots, g_n \in G$ the formula $D_n(x, g_1, \ldots, g_n)$ defines in G the subgroup generated by $g_1, \ldots, g_n$? Surprisingly, there is a wide variety of finitely generated infinite groups with uniformly definable subgroups. Such questions are part of a much bigger problem about the expressive power of the first-order logic in groups (or rings, or arbitrary structures). I will discuss this problem and its connections with the weak second order logic and bi-interpretability.

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