MALTSEV PRODUCTS IN INVESTIGATING LATTICES OF QUASIVARIETIES

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Lattices of quasivarieties which in the literature are called Q-lattices have been investigated for five decades. The investigations have been proposed independently by Garrett Birkhoff and Anatoliĭ Ivanovich Maltsev in [2] and [7], respectively, in the form of the following question: *Which lattices are Q-lattices?*

Many significant results during that period have been achieved and were partially documented in the book [3] and summarized in the survey article [1]. The achieved results had an impact on developing Universal Algebra which was acknowledged in the Epilogue of the book [4] by including the effort of investigating Q-lattices into one out of the five stimulating factors of developing Universal Algebra. The paper [5] is excellent recent evidence of that impact.

Based on the results achieved Q-lattices are either truly complex or tractable. Not much is known about Q-lattices which are at the middle. That is, not much is known about which Q-lattices satisfy a non-trivial lattice identity but which are not distributive. The goal of the talk is to propose a method which suggests a way to fill out this gap. The method is based on the concept of Maltsev product introduced in [6].

Theorem. *For quasivarieties of lattices $\mathcal{K}$ and $\mathcal{L}$, if each of $\mathcal{K}$ and $\mathcal{L}$ satisfies a non-trivial lattice identity, not necessarily the same one, then so does the Maltsev product $\mathcal{K} \circ \mathcal{L}$.*

In the talk, we display applications of this theorem amongst which will be a positive solution of a long standing open problem which is: Is there a quasivariety that is relatively finite-to-finite universal but not Q-universal.

The results presented in the talk are jointly obtained with M.E. Adams or with M.E. Adams and H.P. Sankappanavar.

References

[1] M. E. Adams, K. V. Adaricheva, W. Dziobiak, and A. V. Kravchenko, *Some open questions related to the problem of Birkhoff and Maltsev*, Studia Logica 78 (2004), 357–378.

[2] G. Birkhoff, *Universal algebra*, Proc. First Canad. Math. Congr. (Montreal, 1945), 310- 326, The Univ. Toronto Press, 1946.

[3] V. A. Gorbunov, *Algebraic Theory of Quasivarieties*, Siberian School of Algebra and Logic, Consultants Bureau, New York, 1998.

[4] G. Grätzer, *Universal Algebra*, revised reprint of the 1979 second edition, Springer, New York, 2008.

[5] A. V. Kravchenko, A. M. Nurakunov, and M. V. Schwidefsky, *On representation of finite lattices*, Algebra Universalis, to appear.

[6] A. I. Maltsev, *Multiplication of classes of algebraic systems*, Sibirsk. Mat. Ž. 8 (1967), 346–365.

[7] A. I. Maltsev, *Problems on the borderline of algebra and logic*, Proc. Inter. Congress of Mathematicians, Moscow 1968, 217-231.

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