

*Title: Self-similarity for  $\tau$  groups.*

*Abstract:* Let  $\Gamma$  be a group, let  $\Theta$  be a subgroup and let  $f : \Theta \rightarrow \Gamma$  be a morphism. The  $f$ -core of  $\Theta$  is the biggest subgroup  $C \subset \Theta$  such that  $C$  is normal in  $\Gamma$  and  $f(C) \subset C$ . The pair  $(f, \theta)$  is called a *similar data* for  $\Gamma$  if  $\Theta$  and  $f(\Theta)$  have finite index, if  $\text{Ker } f$  is finite and if the  $f$ -core of  $\Theta$  is trivial. The group  $\Gamma$  is called *self-similar* if it admits a similar data  $(f, \theta)$ . A self similar group acts faithfully on a tree. S. Sidki raised the question if any  $\tau$ -group  $\Gamma$  is self-similar ( a  $\tau$ -group is a finitely generated torsion-free nilpotent group).

Let  $\Gamma$  be a  $\tau$ -group. There is a unique simply connected connected nilpotent Lie group  $N$  such that  $\Gamma$  embeds in  $N$  as a cocompact lattice. Let  $\mathfrak{n}_{\mathbb{R}}$  be the Lie algebra of  $N$ , let  $\mathfrak{n}_{\mathbb{C}} = \mathbb{C} \otimes_{\mathbb{R}} \mathfrak{n}_{\mathbb{R}}$  and let  $\mathfrak{z}_{\mathbb{C}}$  be the center of  $\mathfrak{n}_{\mathbb{C}}$ . In the talk, we will explain the following result:

**Theorem:** *The group  $\Gamma$  is self-similar iff the Lie algebra  $\mathfrak{n}_{\mathbb{C}}$  admits a  $\mathbb{Z}$ -grading:  $\mathfrak{n}_{\mathbb{C}} = \bigoplus_{i \in \mathbb{Z}} \mathfrak{n}_{\mathbb{C}}^i$  such that  $\mathfrak{n}_{\mathbb{C}}^0 \cap \mathfrak{z}_{\mathbb{C}} = 0$ .*