HILBERT'S NULLSTELLENSATZ IN ALGEBRAIC GEOMETRY OVER RIGID SOLVABLE GROUPS

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The classical Hilbert's Nullstellensatz says: if K is algebraically closed field and there is a system of polynomial equations over K, $\{f_i(x_1, \ldots, x_n) = 0 \mid i \in I\}$, then an equation $f(x_1, \ldots, x_n) = 0$ is a logical consequence of this system (satisfies all the solutions of the system in K^n) if and only if some nonzero power of f belongs to the ideal $(f_i \mid i \in I)$ of the ring $K[x_1, \ldots, x_n]$. One can say say that we give an algebraic method for constructing all logical consequences of the given system of equations: f is obtained from f_i $(i \in I)$ using the operations of addition, subtraction, multiplication by elements of $K[x_1, \ldots, x_n]$, and extraction of roots.

Our approach to Hilbert's theorem in algebraic geometry over groups is as follows.

1. We should consider some good class of equationally Noetherian groups, let it be a hypothetical class $\mathcal{K}$.

2. In this class, we need to define and allocate an algebraically closed objects and to prove that any group of $\mathcal{K}$ is embedded into some algebraically closed group. Hilbert's theorem should be formulated and proved for algebraically closed in $\mathcal{K}$ groups.

3. Further, let G be an algebraically closed group in $\mathcal{K}$. We think about equations over G as about expressions $v = 1$, where v is an element of the coordinate group of the affine space G^n.

4. Since an arbitrary closed subset of G^n is defined in general not by a system of equations, but by a positive quantifier free formula (Boolean combination without negations of a finite set of equations) we should consider as basic blocks not equations, but positive formulas.

5. We should specify and fix some set algebraic rules of deduction on the set of positive formulas over G.

6. If the above conditions Hilbert's theorem will consist in a statement that all logical consequences of given positive formula over G are exactly the algebraic consequences.

We realized this approach in algebraic geometry over rigid solvable groups.

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