COUNTING THE BACK-AND-FORTH TYPES

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Given a class of structures $\mathcal{K}$ and $n \in \omega$, we study the dichotomy between there being countably many $n$-back-and-forth equivalence classes and there being continuum many.

In the latter case we show that, relative to some oracle, every set can be coded in the $(n - 1)$st jump of some structure in $\mathcal{K}$. In this case we also show that if $\mathcal{K}$ is $\Pi_2$ axiomatizable, every Turing degree above $0^{(n-1)}$ is the $(n - 1)$st jump degree of some structure in $\mathcal{K}$.

In the former case we show that there is a countable set of infinitary $\Pi_n$ relations that captures all of the $\Pi_n$ information about the structures in $\mathcal{K}$. In most cases where there are countably many $n$-back-and-forth equivalence classes, there is a computable description of them. We will show how to use this computable description to get a complete set of computably infinitary $\Pi_n$ formulas. This will allow us to completely characterize the relatively intrinsically $\Sigma^0_{n+1}$ relations in the computable structures of $\mathcal{K}$, and to prove that no Turing degree can be coded by the $(n - 1)$st jump of any structure in $\mathcal{K}$ unless that degree is already below $0^{(n-1)}$.

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