Comparative analysis of methods for inverse acoustic problems solution

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Let us consider the following problem

\[ c^{-2}(z)v_{tt} = v_{zz} - \frac{\rho'(z)}{\rho(z)} v_z, \quad z > 0, \quad t > 0; \]  
(1)

\[ v|_{t<0} = 0, \quad v_z|_{z=+0} = \delta(t), \]  
(2)

\[ v(+0,t) = f(t). \]  
(3)

Here \( c(z) \geq c_0 > 0 \) \((c_0 = \text{const})\) is the velocity of wave propagation; \( \rho(z) \geq \rho_0 > 0 \) \((\rho_0 = \text{const})\) is the density; \( v(z,t) \) is the acoustic pressure; \( \delta(t) \) is Dirac delta-function.

Forward (direct) problem (1), (2) is to find solution of equation (1) \( v(z,t) \) by known functions \( c(z) \) and \( \rho(z) \).

Inverse problem (1)–(3) is to find functions \( v(z,t), c(z) \) and \( \rho(z) \) by known \( f(t) \).

Let us introduce “travel-time” variable

\[ x = \psi(z), \quad \psi(z) = \int_0^z \frac{d\xi}{c(\xi)}. \]

The function \( \psi^{-1}(x) = z \) exists such as \( c(z) > 0 \) and therefore we can introduce new functions \( u(x,t) = v(\psi^{-1}(x),t), \sigma(x) = c(\psi^{-1}(x))\rho(\psi^{-1}(x)) \).

We reduce (1)–(3) to the following inverse problem:

\[ u_{tt} = u_{xx} - \frac{\sigma'(x)}{\sigma(x)} u_x, \quad x > 0, \quad t > 0; \]  
(4)

\[ u|_{t<0} = 0, \quad u_x|_{x=+0} = c(+0)\delta(t); \]  
(5)

\[ u(+0,t) = f(t), \]  
(6)

where \( v(x,t) = u(z,t) \) is the exceeded pressure; \( \sigma(x) > 0 \) is the acoustic impedance; \( c(+0) \) is known. In order to solve (4)–(6) we have to find functions \( u(x,t) \) and \( \sigma(x) \) by known additional information (6) about the forward problem solution (4), (5).

We consider the following statements of inverse problem (4)–(6): operator, differential, finite-difference, variational and integral.

For recovering unknown coefficient \( \sigma(x) \) in inverse problem (4)–(6) we apply the following methods

- Finite-difference scheme inversion;
- Gel’fand-Levitan-Krein method;
- Boundary Control method;
• Gradient methods (the steepest descent, Landweber iterations);
• Newton–Kantorovich method.

The theoretical and numerical results will be presented and discussed.

References


