ON CAUCHY AND MORERA TYPE CRITERIONS
FOR BOUNDEDNESS OF THE COEFFICIENT
OF DISTORTION

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We give criteria for a mapping to have bounded distortion in terms of an integral estimate of the multiplicity function without any a priori assumption on the differential properties of the mapping. In this talk we extend some results of [1].

Let $\Delta$ be a domain in $\mathbb{R}^n$, $n = 2, 3, \ldots$. Recall that a continuous mapping $f = (f_1, \ldots, f_n) : \Delta \to \mathbb{R}^n$ is a mapping with bounded distortion [2] if the following conditions are fulfilled:

(i) $f \in W_{loc}^{1,n}(\Delta)$;
(ii) The Jacobian $J(f, x) = \det(\frac{\partial f}{\partial x}) \geq 0$ almost everywhere (a.e.) in $\Delta$;
(iii) There exists a constant $K \geq 1$ such that $|f'(x)|^n \leq Kn^{n/2}J(f, x)$ a.e. in $\Delta$, where $|f'(x)| = \left( \sum_{k,l=1}^{n}(\frac{\partial f_k}{\partial x_l})^2 \right)^{1/2}$ is the Hilbert norm of the derivative $f'(x)$. The least constant $K$ is called the distortion coefficient (dilatation) of $f$ [2].

Denote the differential form $f_k \, dx_1 \wedge \cdots \wedge \hat{dx_l} \wedge \cdots \wedge dx_n$ by $\omega_{kl}$. Given a ball $B = B(x, r) \subset \Delta$, consider the numerical $(n \times n)$-matrix

$$\Omega(B) = \left( \int_{\partial B} \omega_{kl}, \ 1 \leq k, l \leq n \right).$$

Endow the space $\mathcal{M}_n$ of all $(n \times n)$-matrices with the Hilbert norm

$$|(a_{kl})| = \left( \sum_{k,l} a_{kl}^2 \right)^{1/2}.$$

**Theorem.** Let $f : \Delta \to \mathbb{R}^n$ be a continuous mapping of a domain $\Delta \subset \mathbb{R}^n$. Then $\exists K_0 > 1$ ($K_0$ doesn’t depend on $f$) such that $f$ is a mapping with distortion at most $K \in [1, K_0]$ if and only if the inequalities

$$\left( \frac{\left|\Omega(B)\right|}{|B|} \right)^n \leq n^{n/2}K^n \frac{\int_{\partial B} N(f|_{B}, y) \, dy}{|B|} < \infty,$$

$$\det \Omega(B) \geq 0$$

hold for every ball $B = B(x, r)$ such that $B(x, 2r) \subset \Delta$.

Here we used the following notations. $|E|$ is the Lebesgue measure of a set $E$, $N(f|_E, \cdot)$ is the multiplicity function of the restriction $f|_E$, i.e., $N(f|_E, y) = \text{card} \left( f^{-1}(y) \cap E \right)$.

**Remark.** Under the extra topological assumption that $f$ is sense-preserving, above Theorem was proved in [1, Theorem 1']. Thus, Theorem 1 is a substantial strengthening of Theorem 1’ of [1].

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