THE RIEMANN – HILBERT PROBLEM
FOR A CLASS OF MODEL VEKUA EQUATIONS
WITH SINGULAR DEGENERATION

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In the talk we consider model equations

$$\frac{\partial v}{\partial z} - \frac{\lambda + \delta|z|^s}{2\pi} v = 0,$$

depending on parameters \( \lambda, \delta \in \mathbb{C} \setminus \{0\} \) and \( s \in \mathbb{N} \), with the singular degeneration at \( z = 0 \) in the region \( G \setminus \{0\} = \{z: 0 < |z| < 1\} \) and with the boundary value condition

$$\Re v|_{\Gamma} = f(t).$$

If \( \lambda = 0 \) this equation is included in the more general class of equations studied by I. N. Vekua [1]. We have proved [6]:

**Theorem.** Let \( \lambda > 0, \delta > 0, s \in \mathbb{N} \) and \( f \in C^{1,\alpha}(\Gamma), 0 < \alpha \leq 1. \) Then the solution \( v \) of this problem is uniquely determined and \( v \) is given by the explicit formula. This solution belongs to the class

$$C(\overline{G}) \cap C^1(G \setminus \{0\}).$$

**Remark 1** (Another known results). This problem is uniquely solvable in the cases

a) if \( \delta = 0 \), and the solution coincide with the Usmanov' solution [4];

b) if \( \lambda \neq 0, \arg \lambda \neq \pi \), and \( \delta \) is small [4];

c) if \( \lambda \) is small, \( \arg \lambda \neq \pi \), and \( \delta \) is arbitrary [5];

d) recently, A. Timofeef and his student from Syktyvkar University have turn up another value of \( \lambda \) and \( \delta \) for which the conclusion of Theorem is true . This paper will by published soon and these results will be in the talk.

In general case (\( \lambda, \delta \in \mathbb{C} \setminus \{0\} \)) we get to explicit formulas of solutions of this equation, depending on parameters \( a_0 \in \mathbb{R}, c_k \in \mathbb{C}, k \in \mathbb{N} \). Finally, we’ve proved that the solvability of this problem is equivalent to existence zeros for the functions \( h_k(\lambda, \delta) \). These functions are given by series.

**Remark 2.** In general case we don’t know the existence of \( \lambda, \delta, k \) such that \( h_k(\lambda, \delta) = 0 \). But from our formulas we can see that

a) The Fredholm alternative take a place, i. e., either the homogeneous problem has a nontrivial solution or the nonhomogeneous problem is solvable for all \( f \);

b) The number \( n \) of linear independent solutions of the homogeneous problem is equal to the number \( n' \) of the linear independent solvability conditions (or \( n = n' < \infty \) or \( n = n' = \infty \)).

Calculation experiments show that functions \( h_k(\lambda, \delta) \neq 0 \) if number \( k \geq N \).
REFERENCES


