## Plenary talks



## Short talks

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### Irreducible representations of finitely generated nilpotent groups

*I.V. Beloshapka (Tomsk)*

We will discuss the proof of Parshin's conjecture, which claims that an irreducible complex representation of a finitely generated nilpotent group is monomial, i.e. is induced from a character of some subgroup, if and only if it is a representation with a finite weight. This was previously known to be true for finite nilpotent groups and for unitary irreducible representations of connected nilpotent Lie groups (A.A. Kirillov and J. Dixmier). We consider representations without any topological structure. As an auxiliary result we will show that for a wide class of induced representations the converse to Schur's lemma holds true. We will discuss examples of non-monomial irreducible representations of the Heisenberg group over the ring of integers.

### Some remarks on quantum logic and a Hilbert space axiom in the orthodox quantum mechanics

*Y.V. Brezhnev (Tomsk)*

This is (some discussion) topic on foundations for nonrelativistic quantum mechanics (QM) and its mathlogical maintanance. There is known Birkhoff's problem of derivation of the Hilbert space lattices (so called quantum logic by Birkhoff-von Neumann (1936)) independently of the Hilbert space axiom in orthodoxal QM. Root of the problem lies in a linearity property of the state space (superposition principle) followed by support with a scalar product structure. We proposed several natural principles (postulats) entailing this superposition principle and scalar product and answering the question why numeric domain in QM should be a complex number set $\mathbb{C}$.

### The Riemann-Roch theorem for the Dynnikov-Novikov discrete complex analysis

*D.V. Egorov (Yakutsk)*

We prove an analog of the Riemann–Roch Theorem for the Dynnikov–Novikov discrete complex analysis.

### Platonic complexities of hyperbolic 3-manifolds

*E.A. Fominykh (Chelyabinsk)*

A triangulation of a 3-manifold M into tetrahedra is minimal if there is no triangulation of M into fewer tetrahedra. The tetrahedral complexity of M is the number of tetrahedra in a minimal triangulation. Similarly we can define cubical, octahedral and dodecahedral Platonic complexities of M. In this talk we calculate Platonic complexities for infinite families of hyperbolic 3-manifolds. The talk is based on a joint work with Colin Adams and Vladimir Tarkaev.

## Capture into parametric autoresonance in non-linear oscillator

*O.M. Kiselev (Ufa)*

We discuss a capture into parametric autoresonance for an equation

$$u'' + (1 + 4\epsilon \cos(\Omega(t,\epsilon)t) \sin(u) = 0, \quad 0 < \epsilon \ll 1. \tag{1}$$

Oscillations of such pendulum with amplitude of order $\epsilon$ can be considered as linear oscillations. Such approach allows us to obtain a resonant frequencies without of any additional calculations. For primary order such frequencies are defined by Mathieu functions. The primary resonance takes place near $\Omega = 2$.

In the resonant interval the solutions grow up to order $\sqrt{\epsilon}$ and the linear approach becomes invalid. Here it is important to study non-linear effects. The growth of the amplitude of solutions up to order $\sqrt{\epsilon}$ is typical for non-linear parametric resonance.

The parametric autoresonance is more sophisticated phenomenon, see [1]. It arises when a phase of the oscillations is captured by a perturbation with slow changing frequency for a lot applications from Faraday waves [2] and plasmas [3] up to quantum phenomena [4]. In the parametric resonance only part of the trajectories can be captured. Two cases are appropriated to study by asymptotic methods. There are solutions of (2) with small amplitude or large amplitude. The oscillations with intermediate amplitudes can be investigated numerically for example.

The amplitude of parametric autoresonance is defined by

$$i\psi' + (\lambda^2\tau - |\psi|^2)\psi + \overline{\psi} = 0, \quad \tau = \epsilon t. \tag{2}$$

This equation defines an evolution of amplitude of non-linear oscillator like (1). This equation can be rewritten in more elegant form of equation for the parametric autoresonance, which looks like as one equation of second order:

$$\varphi'' + 4\lambda^2\tau \sin(\varphi) + 2\sin(2\varphi) - 2\lambda^2 = 0. \tag{3}$$

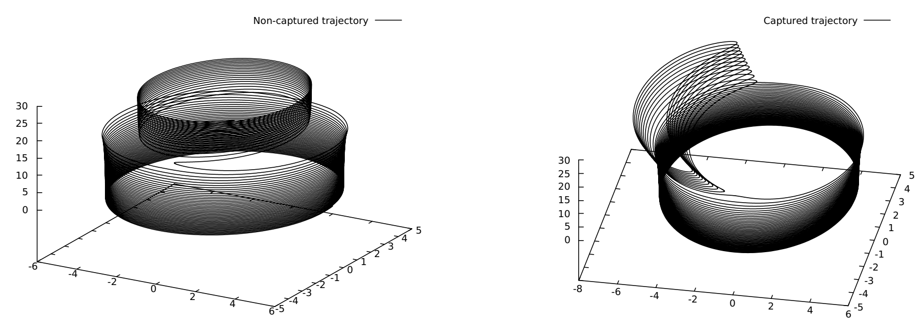


Рис. 1. On the left-hand side one can see a solution of (2) with initial condition $\psi = 5\exp(0.15i)$ at $t = 0$. The trajectory turns at $t \sim 20$. On the right-hand side one can see a solution of (2) with initial condition $\psi = 5\exp(0.19i)$ at $t = 0$. The graph shows how this trajectory is captured at $t \sim 20$. Both trajectories are constructed by Runge-Kutta method of 4-th order with step 0.001.

Our goal is studying of the capture into parametric resonance of large amplitude solutions of (2) as $\tau \to \infty$. It is convenient to investigate such solutions of (2) using a special depending on an inverse value of small parameter:

$$\psi = \varepsilon^{-1}\Psi(\tau, \varepsilon), \quad 0 < \varepsilon \ll 1.$$

Here $\varepsilon^{-1}$ is a parameter of solution, which defines an amplitude of oscillations of $\psi$. After substitution (2) one gets:

$$i\varepsilon^2\Psi' + (\lambda^2\varepsilon^2\tau - |\Psi|^2)\Psi + \varepsilon^2\overline{\Psi} = 0. \tag{4}$$

Note that $\varepsilon$ and $\epsilon$ are small parameters which correspond to different problems. Parameter $\epsilon$ is perturbation of the non-linear oscillator and parameter $\varepsilon$ is a formal parameter which is useful for studying large solutions of (2) and which is derived from (1) for small amplitude oscillations of non-linear oscillator. A condition $\varepsilon \ll \sqrt{\epsilon}$ should be true for an asymptotic formalism which is considered here. The order of amplitudes of oscillations, which are considered here, is *intermediate small:* $\sqrt{\epsilon} \ll |A(\tau)| \ll 1$.

**Theorem 1.** *There exist stable focuses for the equation (3).*

The stable focuses of (3) define the captured solutions. **Theorem 2.** *Beginning at some large $\tau = \tau_0$ up to infinity the measure of captured trajectories has a following asymptotics*

$$M \sim \frac{16\pi\lambda^2}{\tau_0}, \quad \tau_0 \to \infty.$$


[1] E. Khain and B.Meerson. Parametric autoresonance. *Phys. Rev. E*, 64:036619, 2001.

[2] M. Asaf and B. Meerson. Parametric autoresonance of faraday waves. *Physical Review E*, 72:016310, 2005.

[3] J. Fajans, E.Gilson, and L. Friedland. Second harmonic autoresonance control of the $l = 1$ diocotron mode in pure electron plasmas. *Physics Review E*, 62:4131–4136, 2000.

[4] I. Barth and L. Friedland. Quantum phenomena in chirped parametric anharmonic oscillator. *Physical Review Letters*, 113(4):040403, July 2014.


## Lefschetz trace formulas for flows on foliated manifolds

*Y.A. Kordyukov (Ufa)*

In my talk, I will discuss Lefschetz trace formulas for foliated flows on compact manifolds equipped with codimension one foliations. First, I will recall such a formula, due to J. Alvarez Lopez and myself, in the case when the orbits of the flow are everywhere tranverse to the leaves of the foliation. I will briefly describe the role of Lefschetz trace formulas for foliated flows in Deninger's approach to the study of arithmetic zeta-functions. Then I will consider a case when the flow may have fixed points and describe an approach to Lefschetz trace formulas for such flows based on the pseudodifferential b-calculus on manifolds with boundary developed by R. Melrose. I will report on the recent progress in this direction. This is joint work with J. Alvarez Lopez and E. Leichtnam.

## Matrix Integral, Hodge Integral, and Integrable system

*S.-Q. Liu (Beijing)*

Matrix integral is a classical topic in mathematics. It is introduced by physicist E. Wigner, and has many interesting applications in physics, probability theory, mathematical statistics, numerical analysis, and number theory. It is revealed by the celebrated Witten conjecture that matrix integral is also the bridge among two-dimensional quantum gravity, the moduli space of stable curves, and the Korteweg-de Vries (KdV) hierarchy. Hodge integrals are the integrals of certain natural cohomological classes on the moduli space of stable curves, which are very important in modern mathematical physics. In our previous work, we showed that Hodge integral is also related to a certain generalization of the KdV hierarchy. We also conjectured a mysterious relation between matrix Integral and Hodge Integral. Recently, we proved this conjecture. This is a joint work with Boris Dubrovin, Di Yang, and Youjin Zhang.

## The absolute boundary of discrete Heisenberg group

*A.V. Malyutin (St. Petersburg)*

A.M.Vershik introduced the notion of absolute boundary (also called the ‘absolute’) for finitely generated groups. The absolute boundary of a group is a topological space that can be regarded as the boundary at infinity (Dynkin’s exit-boundary) of the so-called dynamical graph over the Cayley graph of the group. The absolute boundary contains, in a sense, the Poisson-Furstenberg boundary of the group and is contained in the Martin boundary of the dynamical graph. A part of the absolute boundary can be identified with the set of all minimal positive eigenfunctions of the Laplacian determined by the simple random walk on the group. The absolute boundary of an abelian group is homeomorphic to a closed ball of certain dimension. (The fact that the absolute boundary of the infinite cyclic group is an interval is a reformulation of de Finetti’s theorem.) The absolute boundary of the free non-abelian group is homeomorphic to the direct product of the Cantor set by an interval. The next phase in developing the theory of absolute boundary is the case of nilpotent groups. We show that in the case of discrete Heisenberg group with the standard generating set, the absolute boundary is homeomorphic to the disjoint union of a closed 2-disk and a countable set of isolated points whose limit set is the boundary of the 2-disk. In order to find the absolute we need, in particular, to describe the set of all geodesic rays in the (Cayley graph of) Heisenberg group.

## Naturally graded Lie algebras and left-invariant geometric structures on nilmanifolds[1]

*D.V. Millionshchikov (Moscow)*

Finite-dimensional positively graded Lie algebras $\mathfrak{g} = \oplus_\alpha \mathfrak{g}_\alpha, \alpha > 0$, are the most elementary examples of nilpotent Lie algebras, furthermore one can show that an arbitrary nilpotent Lie algebra can be obtained as a special deformation of some positively graded Lie algebra. For instance one can consider the filtration $C$ by the ideals $C^i\mathfrak{g}$ of the descending central series and its associated graded Lie algebra $gr_C\mathfrak{g}$ with respect to this filtration.

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[1]The work was supported by RFBR, grant 16-51-55017.

A nilpotent Lie algebra $\mathfrak{g}$ is called naturally graded if it is isomorphic to its $gr_C\mathfrak{g}$.

We will discuss the relation between structure theorems on naturally graded Lie algebras and left-invariant complex structures on corresponding nilmanifolds (we consider Lie algebras with rational structure constantes with respect to some basis).

A left-invariant symplectic (contact) structures on nilfanifolds also closely related to the classification theorems of positively graded Lie algebras (not necessary naturally graded).

### Lagrangian manifolds associated with some problems in abstract analytic number theory

*V.E. Nazaikinskii (Moscow)*

We define the entropy and study typical shapes of elements of an arithmetic semigroup with power-law or exponential asymptotics of the counting function of abstract primes and show how these problems naturally give rise to Lagrangian manifolds and quasi-thermodynamic models. Relations to other fields and possible applications are discussed. The talk is based on results obtained jointly with V. P. Maslov.

### Right-angled Coxeter groups, polyhedral products and hyperbolic manifolds

*T.E. Panov (Moscow)*

Using results on the topology of polyhedral products we describe the structure of commutator subgroups of right-angled Coxeter groups. These results are then applied for diffeomorphism classification of certain hyperbolic manifolds.

### Painleve monodromy varieties: geometry and quantisation

*V.N. Roubtsov (Angers, France)*

We introduce the concept of decorated character variety for the Riemann surfaces arising in the theory of the Painleve differential equations. Since all Painleve differential equations (apart from the PVI) exhibit Stokes phenomenon, we show that it is natural to consider Riemann spheres with holes and bordered cusps on such holes. The decorated character variety is considered here as complexification of the bordered cusped Teichmu?ller space. We also show how to obtain the confluence procedure of the Painleve differential equations in geometric terms. A quantisation and a relation to geometric type cluster algebras.

### Finite dimensional orbits of soliton equations

*T. Shiota (Kyoto)*

A solution of hierarchy of soliton equations having a finite dimensional orbit is often described by a line bundle or a torsion-free rank one sheaf on an algebraic curve. I would like to discuss some aspects of such solutions, including

- study of periodic initial value problem for the K-dV in 1970s - Burchnall-Chaundy-Krichever theory
- Novikov's conjecture and generalizations
- Calogero-Moser-type system for motion of poles of a solution
- Abelian solutions to soliton equations

### Birational automorphisms

*K.A. Shramov (Moscow)*

I will survey various results on groups of birational selfmaps of algebraic varieties, focusing on their finite subgroups. The most interesting examples are provided by birational selfmaps of rationally connected varieties, including classical Cremona groups. In particular, I will explain some recent results concerning Jordan property for these groups.

### Backlund transformations and new bi-Hamiltonian systems on the plane, sphere and ellipsoid.

*A.V. Tsiganov (Moscow)*

Using Jacobian arithmetic for hyperelliptic curves, we can identify well-known cryptographic algorithms and protocols with various schemes of the discretization of continuous Hamiltonian flows in classical mechanics. Most interesting to us fact is that the corresponding auto Backlund transformations also yield new canonical variables on the original phase space, which are useful to construction of new integrable systems in the framework of the Jacobi method. As an example, we show how to construct new bi-Hamiltonian systems with integrals of motion of third, fourth and sixth order in momenta on the plane, sphere and ellipsoid.