Data D can be represented as an attribute-based matrix (x_{ij}), i = 1,…,m; j = 1,…,n; where x_{ij} is the numerical value of j attribute on i-th object. Attributes may be qualitative and quantitative. The fact that numerical values of attributes exist means that there are n measurement procedures, which produce them. Let us denote these procedures as $x_j(a)$, j = 1,…,n, where a is an empirical entity. Then, xij = $x_j(a_i)$.

Attribute-value methods such as neural networks deal with this type of data. These methods are limited by the assumption that data types are stronger than interval and log-interval data types, which is not always true. For definitions of these data types, see Section 4.9.2.

Let us determine an empirical axiomatic system for attribute-based matrices. At first, a set of empirical predicates V_i for each attribute x_I needs to be defined. There are two cases:

1. The measurement procedure x_i is well known and an empirical system is known from measurement theory. Therefore, the set of all empirical predicates contains predicates given in V_i i=1...,n.

2. An empirical system of the measurement procedure x_i is not completely defined. In this case, we have a measurement procedure, but we do not have an empirical system. The measurement procedure in the second case is called a measurer. The examples of measurers are psychological tests, stock market indicators, questionnaires, and physical measurers used in non-physical areas.

Let us define a set of empirical predicates V_i for measurer procedure x_i. For any numerical relation $R(y_1,\ldots,y_k)$ in Re^k (Re - the set of all real numbers), we can define the following empirical relation on A^k.

$$P^R(a_1,\ldots,a_k) \Leftrightarrow R(x_i(a_1),\ldots,x_i(a_k)).$$

The measurer x_i obviously has an empirical interpretation, but relation P^R may not. We need to find such relations R that have empirical interpretations, i.e., relation $R(x_i(a_1),\ldots, x_i(a_k))$ is interpretable in the terms of domain theory.

Suppose that $\{R_1,\ldots,R_k\}$ is a set of the most common numerical relations and some (relations $P^{R1}{}_j,\ldots,P^{Rk}{}_j$) have an empirical interpretation. This set of relations is not empty, because at least the relation $P^{=}{}_j$ (equivalence) has an empirical interpretation:

$$P^{=}{}_j(a_1,a_2) \Leftrightarrow x_j(a_1) = x_j(a_2).$$

In measurement theory, there are many sets of axioms based on just ordering and equivalence relations. Nevertheless, these sets of axioms establish strong data types. A strong data type is a result of interaction of the quantities with individual weak data types such as ordering and equivalence. For instance, having one weak order relation (for attribute y) and n equivalence relations

$$\{\le_y, =_{x_1}, \ldots, =_{x_i}, \ldots, =_{x_n}\}$$

for attributes $x_1,\ldots,x_n$, we can construct a complex relation between y and $x_1,\ldots,x_n$ given by

$$G(y,x_1,\ldots,x_n) \Leftrightarrow y = f(x_1,\ldots,x_n),$$

where $f(x_1,\ldots,x_n)$ is a polynomial [Krantz et al, 1971].

This is a very strong result. To construct a polynomial we need the sum operation, but this operation is not defined for $x_1,\ldots,x_n$. However, relation G is equivalent to polynomial f if a certain set of axioms expressed in terms of order relation ($<_y$) presented above for y, and equality relations (=) are true for x_i.

This fundamental result serves as a critical justification for using ordering relations as a base for generating relational hypotheses in financial applications (Chapter 5). Ordering relations usually are empirically interpretable in finance.

Multivariate and pair comparisons [Torgerson, 1952, 1958, Shmerling D.S. 1978]. Consider set of objects $A = \{a_1,\ldots,a_m\}$ and set of all tuples A^k of k objects from A. A group of n experts are asked to order objects in all tuples $<a_1,a_2,\ldots,a_k>$ from A^k in accordance with some preference relation. Let $a_i{}^{ts}{}_q$ be an object i from tuple $<a_1,a_2,\ldots,a_k>$, where t is an entity to be evaluated, s is an expert and q is a preference rank given by an expert s to the entity t, i = 1,…,m; s = 1,…,n; t = 1,…,$C_m{}^k$; q = 1,…,k. The set of all ordered tuples is denoted by

$$R = \{<a_{i1}{}^{ts}{}_1, a_{i2}{}^{ts}{}_2,\ldots,a_{ik}{}^{ts}{}_k>\}.$$

The typical goal of pair and multivariate comparison methods is to order all tuples. Known methods are based on some a priory assumptions [Torgerson, 1952, 1958; Shmerling D.S. 1978] which determine the areas of applicability.

Let us define for every expert s the preference relation

$$P_s(a_{i1}{}^{ts}{}_{l1}, a_{i2}{}^{ts}{}_{l2}) \Leftrightarrow l1 < l2.$$

Also, define the two equivalence relations ~ , $\sim_t$ and equivalence relation = by

$$a_{i1}{}^{t1s1}{}_{l1} \sim a_{i2}{}^{t2s2}{}_{l2} \Leftrightarrow i1 = i2,$$

$$a_{i1}{}^{t1s1}{}_{l1} \sim_t a_{i2}{}^{t2s2}{}_{l2} \Leftrightarrow t1 = t2, \text{ and}$$

$$a_{i1}{}^{ts}{}_{l1} = a_{i2}{}^{ts}{}_{l2} \Leftrightarrow \text{objects } a_{i1}{}^{ts}{}_{l2}, a_{i2}{}^{ts}{}_{l2} \text{ are the same.}$$

Therefore, we obtain a set of empirical predicates

$$V = \{=, \sim, \sim_t, P_1,\ldots,P_n\}$$

and, thus, a data type represented with the set of empirical predicates V.